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The Unnatural Rate of Unemployment: Reflections on the Barro-Gordon and Natural Rate Paradigms

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Abstract

Unemployment and inflation exhibit a positive correlation during the 1970s in the United States. Ireland (1999) uses the time-inconsistency framework to study long and short run dynamics between the two rates. However, for the long-run, we find that the conditions for cointegration are not met whereas the short-run restrictions grounded in economic theory are strongly rejected. We look at the moving natural rate theory for an alternative explanation of the abnormal behavior of inflation and unemployment. We employ a structural vector autoregression (SVAR) model to study the impact of shocks to natural rate on the two series. We use the Beveridge-Nelson decomposition to extract short-run natural rate estimates from the unemployment series. Further, we identify factors affecting the short-run natural rate using regressions. We conclude that changes in labor-market institutions like unemployment benefits, labor productivity and real wages, as well as changes in labor force growth and real interest rates explain significant variation in the estimated natural rate of unemployment.

Key words: *Phillips curve; Time-inconsistency; Natural rate; SVAR; Beveridge-Nelson decomposition; Labor-market institutions; IV estimation*

JEL Codes: *E31; E52; E61; C22; E24; C3*

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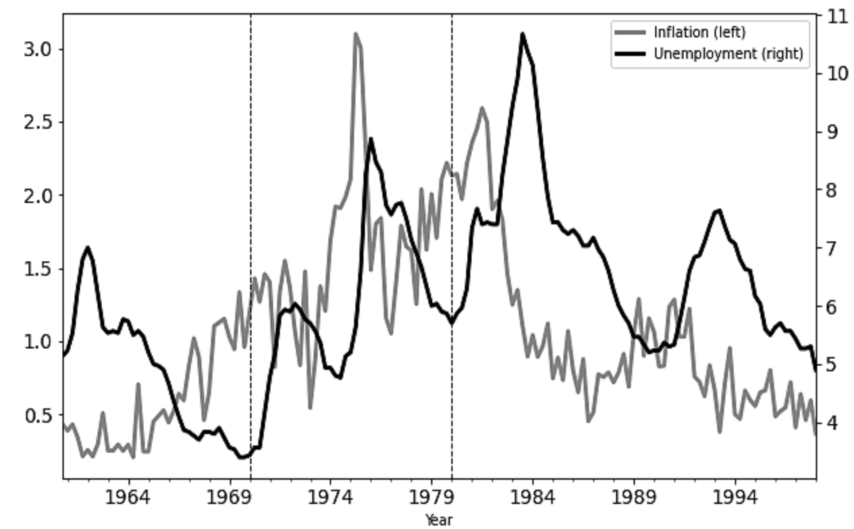
INTRODUCTION

Keynesian prescriptions became ineffective when the Phillips curve broke down in the 1970s. The Phillips curve, as estimated by Samuelson and Solow (1960), was an inverse relationship between unemployment rate and rate of price inflation, tested for the United States. It served to predict that high unemployment should be accompanied by low inflation. In the 1970s, the United States entered an era of stagflation, characterized by simultaneously high unemployment and high inflation. This study attempts to understand factors underlying this newly emerged positive correlation between inflation and unemployment rates by utilizing two sets of theories.

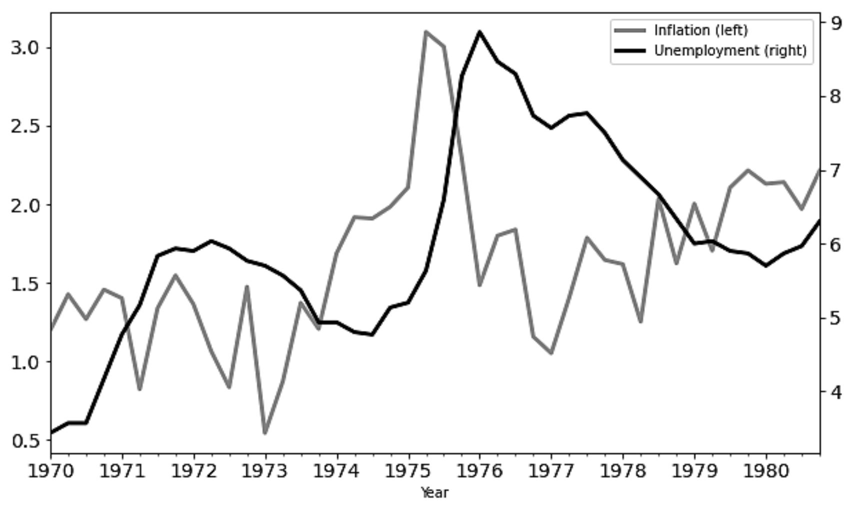
The movement of inflation rate, as measured by quarter-to-quarter percentage changes in the GDP implicit price deflator and the unemployment rate for the period under study is depicted in Figure 1a. The horizontal axis gives the time period in years. The left vertical axis depicts the rate of inflation as changes in deflator while the right vertical axis shows the scale of unemployment rate. The period of interest corresponding to the stagflation period of 1970-1980, marked by two red lines, shows some positive co-movement between the two rates. Figure 1b plots the relationship for 1970-1980. There is a clear positive co-movement around 1974 to 1976.

Figure 1: Inflation and Unemployment Rates Across the Years

a. 1960 – 1997



b. 1970-1980

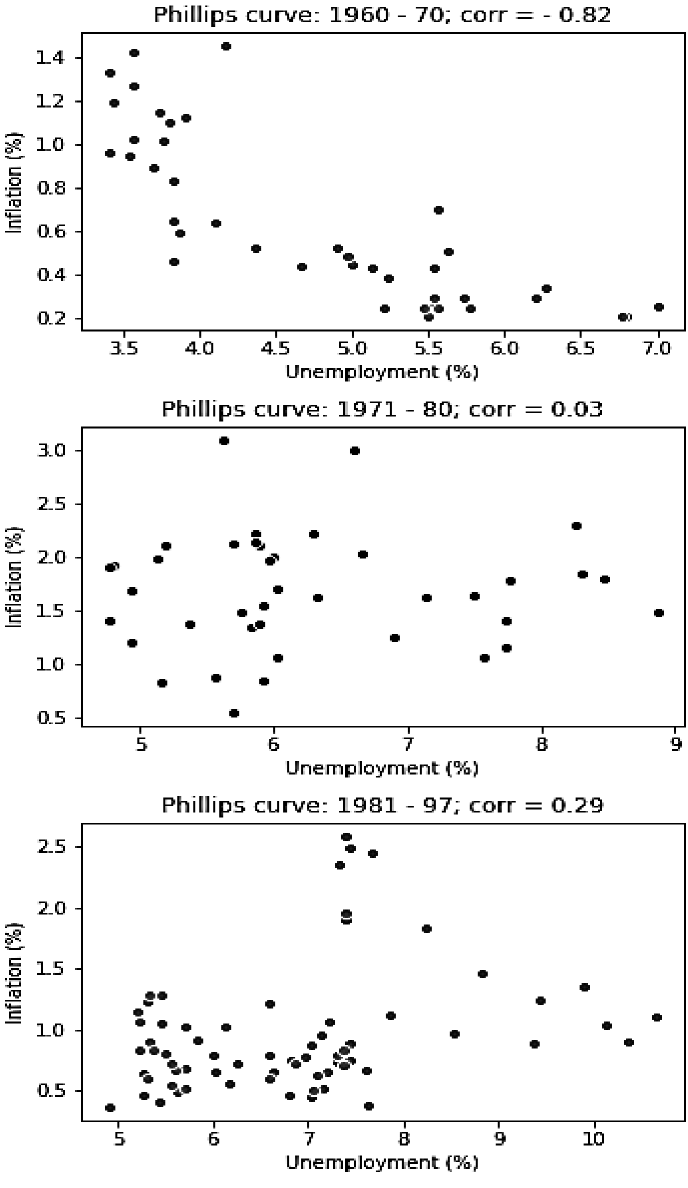


Source: Federal Reserve Bank of St. Louis FRED

To make the visualization concrete, we plotted quarterly rates of inflation and unemployment for segments of the time period considered for this study i.e. 1960 to 1997, in order to better understand the behavior of the Phillips curve as times changed. Figure 2 consists of three graphs for the 1960s, 1970s and 1980s through almost 2000.

The scatter-plot for 1960-70 indicates a significant negative relationship between inflation and unemployment rate, as was characterized by the conventional Phillips curve. The coefficient of correlation between the two rates is -0.82 which signifies that the Phillips curve was quite intact up until this period. The next period's scatter-plot has no discernible pattern and signifies little to no correlation between the two rates, as is also substantiated by the coefficient of correlation which is positive and close to zero. This is proof for what is referred to as "the breaking down of the Phillips curve" in the 1970s when the US experienced high inflation as well as high rates of unemployment. The period of the '80's and '90's sees a mixed relationship between the two rates. At lower levels, there is some positive correlation between the two; however, at higher levels, inflation and unemployment have a somewhat negative though not very significant relation. Across the two decades, we see the correlation at nearly 30 percent which seems to be a reversal of the conventional Phillips curve relation. Now higher unemployment correlates with higher inflation.

Figure 2: The Phillips Curve Across Four Decades



Source: Authors' own calculations

This is when the Keynesian prescriptions of tweaking unemployment in order to achieve target inflation rates by the use of macroeconomic policy came under serious scrutiny. In his seminal contribution famously known as the 'Lucas Critique', Robert Lucas (1976) argued that econometric relationships between variables, which were derived from historical data and may not be structural in nature and could break down if exploited. Parameters of such relationships were a function of the regime and economic policies pursued at that point in time which rendered them fickle for prediction and policy-making purposes. Hence, the Phillips curve disappearing may well have been a consequence of its exploitation.

Two sets of explanations for the stagflation episode were put forth, one rooted in the ideas of rational expectations and the other in the natural rate paradigm. This dissertation will attempt to contextualize the use of these ideas to explain certain features of this period.

Going forward, we set the theoretical foundation by explaining the Barro-Gordon (1983) model which some literature cite as an explanation for the reversal of the Phillips curve. Further, we derive and test the restrictions imposed by the same model and conclude that they fail to explain the long and short-run dynamics between the two rates. We then turn to an alternative explanation in the natural rate paradigm and estimate this rate using the Beveridge-Nelson (BN) (1981) decomposition. By modelling the natural rate as the time-varying steady state of a structural vector autoregression (SVAR), we discuss its dynamic effects on movements in output, inflation and unemployment using Impulse Response Functions and Variance Decompositions. The study is concluded by determining factors responsible for movement of the natural rate itself. We find that changes in labor market institutions such as unemployment benefits, labor productivity and real wages, as well as changes in labor force growth and real interest rates explain significant variation in the natural rate of unemployment.

Rational Expectations – the first paradigm

The idea of rational expectations (RE) was formalized by Muth (1961). It is grounded in the idea that agents in the economy do not waste any information. In a more practical sense, they are aware of the structure of the economy and the nature of variables existing within. Hence, on an average, private agents do not make systematic errors in forecasting. Notwithstanding the apparent ambition in this assumption, there is evidence from several macroeconomic models that rational expectations do hold in the long run.

Taking this idea one step forward, several authors have discussed the idea of optimal policy and social welfare maximization in the light of rational expectations. A highly influential work by Sargent and Wallace (1975) better known as the "policy ineffectiveness proposition" posited how monetary policy could not systematically manage the levels of output and employment in the economy. Kydland and Prescott (1977) similarly argue that a discretionary policy would not result in the maximization of social objectives under this framework. Barro and Gordon (1983) show this argument to hold in a dynamic economic system which they formulate as an econometric model.

Ireland (1999) adapted the Barro-Gordon (B-G) model of time consistent monetary policy to potentially explain the stagflation chapter of the United States; this part of the dissertation seeks to empirically evaluate his findings.

Barro-Gordon (1983) Model of Time-Inconsistency

In the presence of supply side distortions, for example, unemployment benefits, the supply of labor decreases which produces a much lower level of output and a higher level of unemployment. The government is tempted to correct the situation by generating inflation surprises to exploit the Phillips curve. However, such a measure may be stifled due to the problem of time-inconsistency on part of the government. This problem arises when a policy that is optimal to announce today may no longer be optimal to carry out when the time comes to execute it. The reason behind this is that the optimal response of the government is already known to the private agents due to the assumption of rational expectations, thus rendering the monetary policy ineffective. Finally, the equilibrium unemployment level remains unchanged as compared to before while the inflation rate becomes inefficiently high.

Fundamental Relationships of the Barro-Gordon Model

Ireland (1999) modified the B-G model to assess if inflation and unemployment moved together in the long-run and if the trends in inflation over 1960-1997 could be explained by the B-G model. Hence, in this chapter we *test* whether the time-inconsistency problem underlies the behavior of inflation in the United States for the stagflation period. This is to say that an upward trend in the natural rate of unemployment during the 1960s and 1970s allows the B-G model to account for the coincident upward trend in inflation.

The model is based on three fundamental relationships; the expectations-augmented Phillips curve shows the movement of the actual rate of unemployment, U_t around the natural rate, U_t^n in response to deviations of the actual inflation rate, π_t from the expected rate, π_t^e.

$$U_t = U_t^n - \alpha(\pi_t - \pi_t^e) \qquad (1)$$

where $\alpha > 0$.

The natural rate of unemployment, U_t^n is allowed to follow an AR(1) in first differences; this permits the model to treat the actual unemployment rate as non-stationary. The natural rate responds to its past deviations and fluctuates over time in response to a real shock.

$$U_t^n - U_{t-1}^n = \lambda(U_{t-1}^n - U_{t-2}^n) + \varepsilon_t \qquad (2)$$

where $-1 < \lambda < 1$ and $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is serially uncorrelated.

Lastly, we have the relationship between the realized and expected inflation rates, where π_t^p is the planned rate of inflation chosen by the government after the private agents form their expectations for inflation. This model introduces control errors for inflation η_t which permits accounting of the transitory deviations between the actual unemployment rate and the natural rate.

$$\pi_t = \pi_t^p + \eta_t \tag{3}$$

where $\eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$ is serially uncorrelated and has covariance $\sigma_{\eta\varepsilon}$ with ε_t.

Solving the Model

The policymaker chooses the optimal π_t^p to maximize its policy objectives around the target values of unemployment and inflation. Since target unemployment must be lower than realized unemployment, it is given by kU_t^n for all $0 < k < 1$ and inflation is targeted at zero so that $\pi^* = 0$.

This returns the government's loss function which penalizes variations of unemployment and inflation from their targets.

$$(1/2)(U_t - kU_t^n)^2 + (b/2)\pi_t^2 \tag{4}$$

where $b > 0$ and is the relative weight assigned to unemployment stabilization vis-à-vis inflation stabilization.

We want to solve for π_t^p, π_t and U_t. The reader may refer to Appendix for the complete calculations. Using the rational expectations hypothesis (**RE**), the inflation and unemployment rates are functions of the natural rate of unemployment; we get the *equilibrium* expressions of the two rates as

$$U_t = U_{t-1}^n + \lambda \Delta U_{t-1}^n + \varepsilon_t - \alpha\eta_t, \tag{5}$$
$$\pi_t = \alpha A U_{t-1}^n + \alpha A \lambda \Delta U_{t-1}^n + \eta_t \tag{6}$$

Since the underlying natural rate is assumed to be non-stationary in this model, equations (5) and (6) imply that π_t and U_t are also non-stationary processes.

We are interested in the *long-run relationship* of the two variables which can be tested through a test for *cointegration*. The necessary conditions for cointegration are:

- The series must be integrated of order d for all $d \geq 1$
- Their linear combination must be integrated of order less than d

If the two series are cointegrated, their long-run relationship can be inferred from the model. To understand if the model can explain the short-run *dynamic relationships* (quarter-to-quarter co-movement) between these two variables, we also impose certain short-run restrictions on the two rates.

Derived Relationships

Equilibrium relationships determined by the model impose certain structure on the behavior of the state variables. These can be summarized as long and short run restrictions that the model imposes on the same variables. If the data shows that these restrictions are significant then we believe that model is a good representation of the actual relationships.

The model imposes the *long-run constraint* of cointegration on the bivariate time-series of inflation and unemployment to test if the two series move jointly in the long run. This is evident from the simultaneously existing positive relationships of the two series with the natural rate as an anchor. A linear combination of the two series makes this constraint

$$\pi_t - \alpha A U_t = -\alpha A \varepsilon_t + (1 + \alpha^2 A)\eta_t, \tag{7}$$

According to the model, this linear combination is a stationary process as it is a function of white noise. If data can support stationarity for this relationship along with non-stationarity for the bivariate series, then the model would have successfully explained the cointegrating relationship between inflation and unemployment rates.

A vector ARMA (1, 2) of the cointegration constraint and the first difference of unemployment (for stationarity) is constructed. The within-equation and cross-equation restrictions in the vector ARMA summarize the *short run constraints* imposed on the bivariate series by the model. Theory places a total of 10 restrictions on this model as is clear from the constants in the coefficient matrices. It is given by

$$\begin{bmatrix} \pi_t - \alpha A U_t \\ \Delta U_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} \pi_{t-1} - \alpha A U_{t-1} \\ \Delta U_{t-1} \end{bmatrix} + \begin{bmatrix} -\alpha A & 1 + \alpha^2 A \\ 1 & -\alpha \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \alpha(1+\lambda) \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1} \\ \eta_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\alpha\lambda \end{bmatrix} \begin{bmatrix} \varepsilon_{t-2} \\ \eta_{t-2} \end{bmatrix} \tag{8}$$

If these restrictions are statistically significant for the given data, then they hold true and the model would successfully explain the short-run dynamics between the inflation and unemployment rates.

For complete derivations of the restrictions, refer to Appendix.

Data sources

Data has been drawn from the Federal Reserve Bank of St. Louis' FRED database, and is available in a quarterly and seasonally-adjusted form. The inflation rate is measured using quarter-to-quarter percentage changes in the GDP implicit price-deflator. The unemployment rate is measured using the civilian unemployment rate.

Empirical Strategy and Estimates

Long run dynamics

The model treats inflation and unemployment rates as non-stationary. We test this model characteristic by conducting some tests- Kwiatkowski–Phillips–Schmidt–Shin (KPSS), Augmented Dickey Fuller (ADF) and Phillips Perron (PP) test, which check for unit roots in both series; the results are reported in Table 1. The null hypothesis of the PP and ADF tests is that the series contains a unit root and is therefore, non-stationary. Rejection of the null implies that the given series is trend stationary. For KPSS test, the null and alternate hypotheses are reversed. A more detailed table for the PP-test with persistence parameters is given in Appendix as Table B.1.

Table 1: Unit Root Tests

Sample period	KPSS t-stat	ADF t-stat	PP t-stat
1960:1 - 1997:2			
Unemployment rate	1.3134	-0.3746	-2.2161
Inflation rate	2.5151	-1.4366	-2.8739**
1974:1 - 1997:2			
Unemployment rate	0.5206	-0.3071	-2.1205
1981:4 - 1997:2			
Inflation rate	0.2847	-2.0845**	-4.3842**

Note: ** Denotes significance at the 5 percent level.

The unit root tests fail to reject the null hypothesis of unit root for unemployment in all samples and reject the null for inflation; thus, unemployment contains a unit root and exhibits non-stationarity while inflation exhibits trend stationarity.

Figure 1 suggests a positive co-movement between inflation and unemployment. However, the cointegration hypothesis cannot be tested for the two series since one of them i.e. inflation, is stationary. Nevertheless, a simple test of cointegration was conducted on the bivariate series to eliminate any doubt and the null hypothesis of 'no cointegration' overwhelmingly failed to be rejected with a p-value of 0.23. Ireland (1999) may have concluded that both series were non-stationary since he did not account for the appropriate structural breaks with econometric testing, rather an arbitrary break in 1970. W.F. Mitchell (1993) discusses that the widespread acceptance of the unit root hypothesis arises because of misspecification in the testing regression, specifically due to failure to account for segmented trends. In general, the ADF and PP tests have very low power against I(0) alternatives that are close to being I(1). That is, unit root tests cannot distinguish highly persistent stationary processes from non-stationary processes very well.

We find a structural break in the series of unemployment at 1974Q1 (Papell et. al. 2000) and inflation at 1981Q4; to account for these breaks, sub-samples have been derived from the full sample and tests are conducted on all samples. Our results suggest that the Barro-Gordon model, as modified by Ireland (1999), cannot establish a long-run relationship between inflation and unemployment since its long run constraints were rejected by data.

Short run dynamics

We now turn our focus on the theory's implications for the short-run behavior of inflation and unemployment i.e. to test for quarter-to-quarter co-movement between the two rates.

Table 2: Maximum Likelihood Estimates

Parameter	Estimate	Standard error
α	0.1475	0.0566
A	1.1549	0.4440
λ	0.5686	0.0681
σ_{ε}	0.2675	0.0159
σ_{η}	0.6299	0.0368
$\sigma_{\varepsilon\eta}$	0.0573	0.0148

Notes: L^c: maximized value of constrained log-likelihood = -148.6597
L^u: maximized value of unconstrained log-likelihood = -11.22
LR = $2(L^u - L^c)$ = 274.8750
Critical value for LR with 10 dof at 0.1 percent significance = 29.6

Table 2 records the maximum likelihood estimates of the model's parameters which are obtained by mapping the constrained vector ARMA(1,2) from equation (8) into state-space form and then using the Kalman filter algorithm to evaluate the likelihood function by filtering.

The general premise of a state space model is that we define a system by a set of states (a state is a vector containing relevant information about the system at some point in time) that evolve in time, but our observations of these states contain statistical noise, and hence we are unable to ever directly observe the "true" states. The goal of the state space model is to infer information about the states, given the observations, as new information arrives. A famous algorithm for carrying out this procedure is the Kalman Filter. For our case, we observe the inflation and the unemployment rate, while the short term natural rate is the unobserved state.

We find that the parameter estimates are stable and fairly accurate as the standard errors are small in magnitude. From the first row of estimates, we find that α is approximately 0.15 across the board, implying that each percentage point of surprise inflation generated a 0.15 percentage point fall in the unemployment rate. This suggests that the Phillips curve is quite steep.

The second row shows that the estimate of A exceeds unity. Given the constraint $0 < k < 1$, b must be less than 1 i.e. $0 < b < 1$ (since $A = (1 - k)/b$, see Appendix). This coefficient implies that the Fed placed more weight on unemployment stabilization as compared to inflation stabilization, over the sample period. The positive estimates of the covariance term $\sigma_{\varepsilon_\eta}$ suggest that unfavorable shocks to the natural rate tend to coincide with unfavorable shocks to inflation.

Now, to test the hypothesis that the short-run constraints hold, we have a constrained and an unconstrained vector ARMA(1,2) model each; we test for the within and cross-equation restrictions by comparing the fit of these two models. The constrained model has 6 parameters while the unconstrained ARMA(1,2) state-space model has 16 parameters. Theory places 10 restrictions on the bivariate time-series model. From the LR statistic values we can conclude that the likelihood ratio tests overwhelmingly reject the model's restrictions. i.e. the constrained model does not hold in the short-run.

The model was unable to successfully account for the dynamic, quarter-to-quarter co-movement of inflation and unemployment in the US for the given four decades. This may be due to a large number of restrictions and a highly simplified model by the theorists. Ireland (1999) also finds a high degree of serial correlation (persistence) in the dynamic relationship between inflation and unemployment and cites that as a potential reason for the rejection of the short-run model.

Remarks on the effectiveness of the paradigm

In conclusion of this section, we conducted an exercise based on Ireland (1999) to test whether the time-inconsistency problem as described by Barro and Gordon (1983) can explain the behavior of inflation in the United States for the period largely encompassing the stagflation episode. From the results presented above, we conclude that our hypothesis from the beginning of this chapter is rejected, since both long run and short run relationships between the positively correlated inflation and unemployment series during this time could not be explained by the theoretical underpinnings of the Barro-Gordon model of time consistent monetary policy. We could not establish a long-run relationship of co-movement between inflation and unemployment using the concept of cointegration since the two were not I(d) series and one of them viz. inflation was even I(0). This renders the basis of cointegration testing null and void. We could also not establish that the two series exhibited co-movement between quarters i.e. in the short run. This may have been because the theory places several restrictions on the short-run dynamics of the two series or due to the presence of auto-correlation in their linear combination or both. A useful takeaway from Ireland's (1999) work is that the natural rate positively drives both inflation and unemployment rates.

Natural Rate – the second paradigm

The period of stagflation in the United States was fertile ground for several macroeconomic thoughts to grow as alternatives to Keynes, one of which was the Natural Rate Hypothesis (NRH).

Propounded by Milton Friedman (1968) and Edmund Phelps (1967) simultaneously, the natural rate, also known as the 'Non-Accelerating Inflation Rate of Unemployment' (NAIRU) is the rate of unemployment consistent with stable inflation. This rate is ground out by the set of "Walrasian" microeconomic relations in the economy which are rooted in actual structural characteristics of the labor and commodity markets such as strength of labor unions, cost of gathering information about job vacancies and labor availability, to name a few.

The natural rate theory implies that unemployment fluctuates around its natural rate in the short run. Since firms and workers care about real wages over nominal wages, their "expectation" of wage and/or price inflation becomes written into wage contracts. If they accurately forecast the wage inflation, their subsequent actions to maintain real wage levels would equate the unemployment rate with its steady-state level. However, the short-run fluctuations around the natural rate derive from realized errors in forecasting this very rate of wage inflation. Hence, fluctuations around the natural rate arise due to the presence of nominal rigidity (slow adjustment of nominal wages or price stickiness) and demand shocks (monetary and fiscal disturbances).

The primary implication of the natural rate theory is that demand disturbances do not have any long run effects on the real economy since the natural rate is a variable independent of demand side policies and shocks. This is the idea of long-run neutrality of money. Hence, a reduction in unemployment induced by a higher level of inflation would be short-lived and the unemployment rate would eventually revert to its natural rate.

Although the natural rate is independent of demand shocks, it has been shown to shift due to productivity or supply shocks, particularly owing to changes in labor market institutions. Phelps (1994) argues that the natural rate of unemployment is the current equilibrium steady state of the unemployment rate which is not a constant or exogenous force, rather the endogenously determined moving target which the equilibrium unemployment path constantly pursues.

Phelps (1994) also finds that for most OECD countries, the unemployment rate is stationary around its moving target of the natural rate. When the structural breaks are accounted for, the persistence level in the unemployment series falls by a sizeable 0.30; the resulting coefficient of persistence then implies fairly rapid mean reversion for unemployment rate. Hence, Phelps (1998) argues that the unemployment problem may often be described as a shift in mean unemployment rather than high persistence. If most shocks cause temporary movements of unemployment around the natural rate, but occasional shocks cause permanent changes in the natural rate itself, unemployment would be stationary around a process that is subject to structural breaks (Papell et. al. 2000). Papell also finds strong evidence of trend-stationarity of the unemployment process after incorporating structural change, which is more evidence for disproving the high persistence (or hysteresis) theory of unemployment.

Fair (2000) has demonstrated that high persistence in unemployment makes it difficult to reject a unit root in practice. This might explain why Ireland (1999) saw non-stationarity in the unemployment process; his failure to account for the true structural break may have resulted in a high persistence estimate and an incorrect inference.

Identifying the Natural Rate

We have established from literature that the unemployment rate is determined by a stable dynamic process and, in the absence of exogenous shocks, converges to a unique steady-state equilibrium which is the natural rate. This equilibrium is itself endogenous and is a moving target pursued by the unemployment rate. Within this framework, King and Morley (2007) estimate the natural rate of unemployment using a structural VAR model of aggregate output, inflation and unemployment.

We follow a similar methodology[1] and use a SVAR model to explore the impulse responses of the state variables to the exogenous structural shocks and decompose the variances to evaluate the relative importance of each shock for variation in the state variables. We also estimate the short-run natural rate using the Beveridge-Nelson (1981) decomposition method.

1. Structural Vector Autoregression

Vector autoregressive (VAR) models have made multivariate data analysis possible while enabling the models to incorporate well grounded theoretical knowledge as a priori information, to enrich dynamic relationships between variables in time. These models explain the endogenous variables by only their own history (lags), apart from certain deterministic regressors. However, often contemporaneous interdependence might exist between the endogenous variables of the system. structural vector autoregressive (SVAR) models allow for explicit modeling of relationships between endogenous variables that may be dependent on lags of each other. A SVAR model can be used to identify shocks and trace them out by employing impulse response functions (IRF) and forecast error variance decomposition (FEVD).

Typically, a VAR(p) process is defined as

$$y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t$$

where A_i are coefficient matrices for lags $i = 1, .., p$ and $E(u_t) = 0$. Its coefficients can be efficiently estimated using ordinary least squares (OLS).

---

[1] King and Morley (2007) conduct their study for a longer time period: 1948Q2 : 2001Q1.

On the other hand, a SVAR(p) model is the structural form of the reduced-form VAR(p), and is defined as

$$Ay_t = A_1^* y_{t-1} + \ldots + A_p^* y_{t-p} + B\epsilon_t$$

where A_i^* are structural coefficient matrices for lags $i = 1, .., p$ and the structural errors, ϵ_t are characterized by white noise.

If forecasting is the objective of multivariate modelling, estimation of the parameters of the VAR model is sufficient. However, in order to explore the evolution of the structural relationship between the endogenous variables, one needs to identify the parameters of the structural equation given above. Its parameters are estimated by minimizing the negative of the log-likelihood function. In both models, y_t is a vector of endogenous variables which is assumed to be stationary.

In between transforming the structural equation (SVAR) to its reduced form (VAR) for estimation, the number of structural parameters exceeds the number of equations in the reduced form. In order to identify the entire system, some structural parameters have to be restricted. In general, for a SVAR(k) model, $k(k-1)/2$ restrictions must be imposed for identification.

SVAR identification

We want to determine the short-run effects of changes in the natural rate on unemployment and inflation in particular, in order to study the positive relationship established between the pairs in the previous chapter. As we discussed, this is possible with the use of IRF and FEVD in a SVAR model.

Let us consider the vector,

$$\mathbf{x_t} = [\mathbf{y_t} \quad \mathbf{p_t} \quad \mathbf{u_t}]$$

where $\mathbf{y_t}$ is column vector of real GDP, $\mathbf{p_t}$ is the column vector of consumer price index and $\mathbf{u_t}$ is the column vector of unemployment rate in quarter t. We describe the reduced-form dynamics of the first differences of these series by a stationary VAR model[2]:

$$\Delta\mathbf{x}_t = \mathbf{c} + \sum_{k=1}^{K} \mathbf{F}_k\, \Delta\mathbf{x}_{t-k} + \mathbf{e_t}$$

where $\mathbf{c}$ is a vector of constants, $\mathbf{F_k}$ is a matrix of coefficients, and $\mathbf{e_t}$ is a vector of normally distributed forecast errors with zero mean.

The structural model (SVAR) can be represented by an infinite-order moving-average process given by

$$\mathbf{B}\Delta\mathbf{x}_t = \mathbf{m} + \sum_{k=0}^{\infty} \mathbf{A}_k\, \mathbf{v}_{t-k}$$

where $\mathbf{B}$ is the coefficient matrix for $\mathbf{x}_t$, $\mathbf{m}$ is a vector of deterministic drifts for the endogenous levels variables in $\mathbf{x}_t$, $\mathbf{A}_k$ is the matrix of shock coefficients, and $\mathbf{v}_t$ is a vector of three structural shocks, $\epsilon_{yt}, \epsilon_{pt}$ and ϵ_{ut}. The shocks are assumed to satisfy conditions of stationarity, have means of zero, variances of unity and zero cross-correlations.

The SVAR system in infinite-order MA form helps us make useful inferences by focusing on the disturbances over the long-run. In matrix algebra form, it is given by

$$\begin{bmatrix} 1 & b_{12} & b_{13} \\ b_{21} & 1 & b_{23} \\ b_{31} & b_{32} & 1 \end{bmatrix} \begin{bmatrix} \Delta y_t \\ \Delta p_t \\ \Delta u_t \end{bmatrix} = \begin{bmatrix} \mu_y \\ \mu_p \\ \mu_u \end{bmatrix} + \sum_{k=0}^{\infty} \begin{bmatrix} a_{11,k} & a_{12,k} & a_{13,k} \\ a_{21,k} & a_{22,k} & a_{23,k} \\ a_{31,k} & a_{32,k} & a_{33,k} \end{bmatrix} \begin{bmatrix} \epsilon_{y,t-k} \\ \epsilon_{p,t-k} \\ \epsilon_{u,t-k} \end{bmatrix}$$

Let us see how to interpret the coefficients of matrix $\mathbf{A}_k$ from the above representation. $a_{11,k}$ captures the impact of a shock to output on

[2] By taking first differences and modelling a stationary VAR, we follow King and Morley (2007) in assuming that all three endogenous levels variables are I(1) processes.

change in output in period k. The aggregate effect of output shocks to change in output over the long run (infinite horizon) would then be given by $\sum_{k=0}^{\infty} a_{11,k}$.

To determine short-run effects of structural shocks on the system of endogenous variables, we first need to fully identify the system, and to that end we will follow Blanchard and Quah (1989) by imposing long-run identifying restrictions on the relationship between the observable data and the structural shocks. Firstly, we are interested in estimating effects of shocks on each endogenous variable individually; hence, we restrict the $\mathbf{B}$ matrix to an identity matrix by setting all b_{ij} coefficients equal to 0. Further, we impose the following identifying restrictions on the system:

$$\sum_{k=0}^{\infty} a_{31,k} = 0, \qquad \sum_{k=0}^{\infty} a_{32,k} = 0, \qquad \sum_{k=0}^{\infty} a_{12,k} = 0$$

We can map these coefficients back to the system of equations in (3.3) for better understanding of the restrictions. From (3.4), the restrictions impose that:

1. the first structural shock has no long-run effect on the unemployment rate: $a_{31,k}$
2. the second structural shock has no long-run effect on both, the unemployment rate: $a_{32,k}$, and on output: $a_{12,k}$

From Blanchard and Quah (1989), our first structural shock, ϵ_{yt} can be thought of as an aggregate supply (or productivity) shock (AS) and the second structural shock, ϵ_{pt}, can be thought of as an aggregate demand (or money supply) shock (AD). Our third structural shock, ϵ_{ut}, is completely unrestricted. Since this is the only shock that is allowed to have long-run effects on the unemployment rate, we will call this the natural rate (NRU) shock. Restrictions on the AD shock represent the concept of long-run monetary neutrality, in that none of the real variables are nominally affected in the long run.

Results for the SVAR Model

a. Impulse Response Function

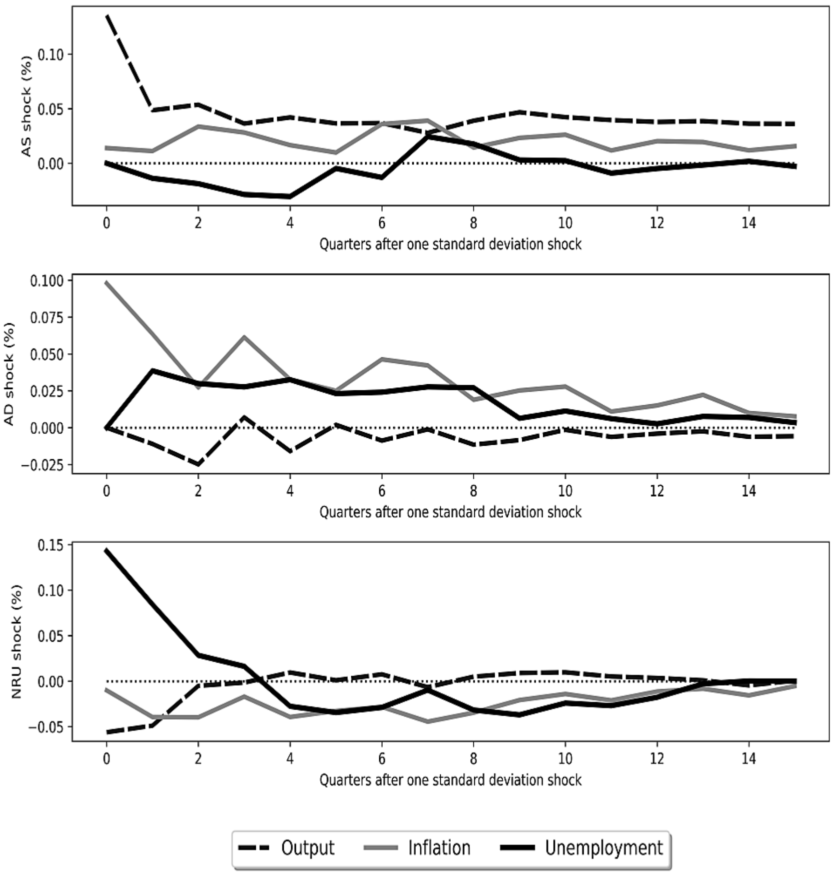
We are interested in determining how our endogenous variables evolve over a specified time horizon when an exogenous shock is introduced. This insight is given by impulse response functions. The way it works is that for each variable from each equation separately, one standard deviation shock is applied to the error, and the effects upon the VAR system over time are noted. If the system is stable, the shock would gradually die away.

Note that only the NRU shock is allowed to impact the unemployment in the long-run in our model, by construction. We estimate the IRFs of all three endogenous variables to each of the shocks to see their responsiveness over 15 quarters since the introduction of one unit shock (Figure 3). A one standard deviation positive AS shock structurally increases the level of output by approximately 0.05 percent and has no effect on unemployment, also by construction. The AD shock has no structural effect on any real variables, as our theory also claims.

Finally, we see that the unemployment series does not have any long-run effect due the NRU shock as well. After about 13 quarters of one standard deviation shock in unemployment, its effect on unemployment dissipates entirely. This result can be held as testament to our original assumption of the moving natural rate theory wherein unemployment was supposed to be a stationary process which fluctuated around the non-stationary moving natural rate. The IRF for unemployment suggests that there is no long run persistence of shocks[3].

[3] This result is in contrast to King and Morley (2007), who find that the NRU shock has a permanent effect on the unemployment rate of 0.4 percent.

Figure 3: Impulse Response Functions



Source: Authors' own calculations

However, results from these IRFs must be taken with a pinch of salt. Conceptually, IRFs allow us to check for dynamic relationships after a one standard deviation shock (1-SD) to one of the variables. However, since our series are in first differences, this definition breaks down as a 1-SD shock in differences doesn't have much practical explanation. That is why Sims (1980) suggests applying SVAR on level series to check for dynamic relationships between the series. However, we must note that the estimated coefficients will not be stable for hypothesis testing when our SVAR model is non-stationary.

We then estimated IRFs of our endogenous variables in level form as well as by making the time-series stationary.[4] Graphical IRFs of unemployment to the NRU shock for both cases have been given in Appendix as Figures C.1 and C.2. It is worth noticing that unemployment almost always reverts to its pre-shock level after a shock in NRU is introduced. The reversion happens faster when the unemployment series has been treated in some way for stationarity, around the thirteenth quarter; the level series converges to its pre-shock level after about 30 quarters. However, while the standard errors for our stationary series' IRFs are fairly limited, these bands are ever-increasing in the IRFs for level series. This places relatively more confidence in the estimates derived from series that are stationary. These results suggest that though the NRU shock has some effect on the unemployment rate in the medium-run, in the long-run, unemployment is not significantly influenced by structural shocks.

b. Variance Decomposition

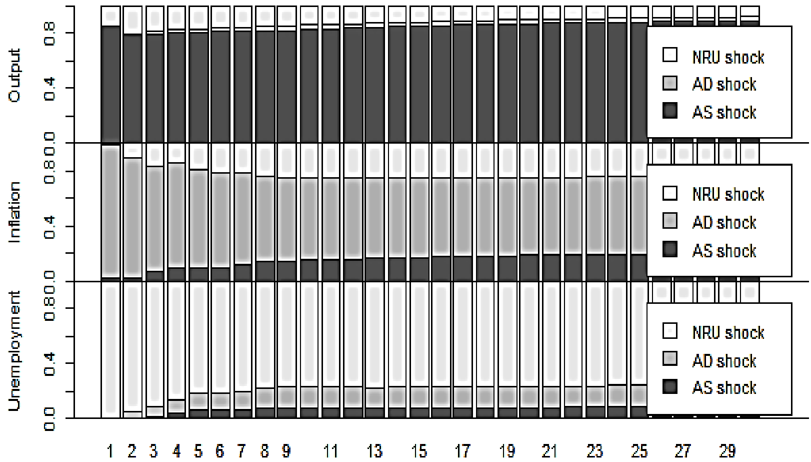
We explore a slightly different method for examining VAR system dynamics by using forecast error variance decompositions (FEVD). They give the proportion of movements in the endogenous variables that are due to their 'own' shocks, versus shocks to the other variables. This decomposition is useful because while a shock to the i^{th} variable will directly affect itself, it will also be transmitted to all of the other variables in the system through the dynamic structure of the VAR.

In practice, it is usually observed that own series shocks explain most of the (forecast) error variance of the series in a VAR. The same holds for our model; AS shocks dominate output, AD shocks dominate

[4] King and Morley (2007) modified output and inflation by taking first differences of their logs. Since using the same methodology yielded integrated series for our data, we take the first diff. of u_t, and second diff. of y_t and p_t.

inflation and NRU shocks dominate movements in unemployment n-periods after a 1-SD structural shock has been introduced. The variance decompositions are depicted in Figure 4.

Figure 4: Variance Decompositions of 30-Period Ahead Forecast Errors



Source: Authors' own calculations

The NRU shock explains between 75-98 percent of the movement in unemployment rate at most time horizons, $n \leq 30$, and explains 15 percent and 9 percent of the movements in inflation and output respectively. Supply side shocks explain over 80 percent of the variation in output and explain hardly 3 percent movement in inflation and 20 percent in unemployment. As for the demand side disturbances, they dominate variation in inflation between 55-96 percent and explain 20-25 percent of variation in output and unemployment respectively in the short run.

2. Beveridge-Nelson Decomposition

Now that we have determined the extent to which structural shocks impact our endogenous variables in the short and long run, we can say that shocks in the natural rate explain the most part of movements in the unemployment rate. Our overarching objective is to understand the unconventional movement in unemployment and inflation rates during the US stagflation episode. To this end, we need an estimate of the natural rate for the same period of time. Since the natural rate is a theoretical construct and hence unobservable in real time, we will estimate it as the trend component of the unemployment time-series.

Structure

A large number of studies have shown that many economic time series are well represented by the class of ARIMA$(p, 1, q)$ processes. They are non-stationary processes for which the first differences are a stationary process of ARMA(p, q) form. Let $\mathbf{z_t}$ contain a unit root:

$$z_t = z_{t-1} + u_t$$

where u_t is not a white noise process, but an ARMA(p, q) process.

In their original paper, Beveridge and Nelson (1981) noted that it is possible to decompose an integrated process into its trend (permanent) and cyclical (transitory) components. In particular, $\mathbf{z_t}$ can be decomposed into two components:

$$z_t = \tau_t + \xi_t$$

where τ_t is the trend and is a pure random walk, i.e. $\tau_t = \tau_{t-1} + \epsilon_t$, where ϵ_t is a white-noise process. The BN estimate of trend for an I(1) time series $\mathbf{z_t}$ is defined to be the limiting forecast as the horizon goes to infinity, with its slope equal to the rate of drift of the series, or the mean growth rate. The cyclical component given by ξ_t is a stationary process which represents the sum of forecastable future changes in $\mathbf{z}$ at time t but is expected to be dissipated as the series tends to its trend or permanent level.

Estimating the Natural Rate from BN-Trend

Given a reduced-form time series model, the steady state of a series can always be estimated using the BN decomposition. Given the moving natural rate theory, we will estimate the natural rate as the steady state of the unemployment series which we know to be non-stationary, a necessary condition for carrying out the decomposition. Let us set up a model where the integrated unemployment series, $\mathbf{u_t}$, can be modestly forecast using a stationary univariate AR(1) model for its first differences:

$$(\Delta u_t - \mu) = \phi(\Delta u_{t-1} - \mu) + \epsilon_t$$

where μ is the long-run mean of the differenced u_t series, $|\phi| < 1$ and $\epsilon_t \sim i.i.d\mathcal{N}(0, \sigma^2)$. We are interested in the estimation of the BN trend component of $\mathbf{u_t}$. The BN trend (BN_t^τ) is defined as the minimum mean squared error (MSE) forecast of the long-run level of the series (minus the deterministic drift), when the periods indexed by j tend to infinity:

$$BN_t^\tau = \lim_{j\to\infty} E_t[u_{t+j} - j.\mu]$$

Equivalently, the trend can also be defined as the present level of the series plus the infinite sum of the minimum MSE, j-period ahead, first difference (mean deviation) forecasts:

$$BN_t^\tau = u_t + \lim_{J\to\infty} \sum_{j=1}^{J} E_t\left[\Delta u_{t+j} - \mu\right]$$

Hence, the BN trend of $\mathbf{u_t}$ for a univariate AR(1) process is given by:

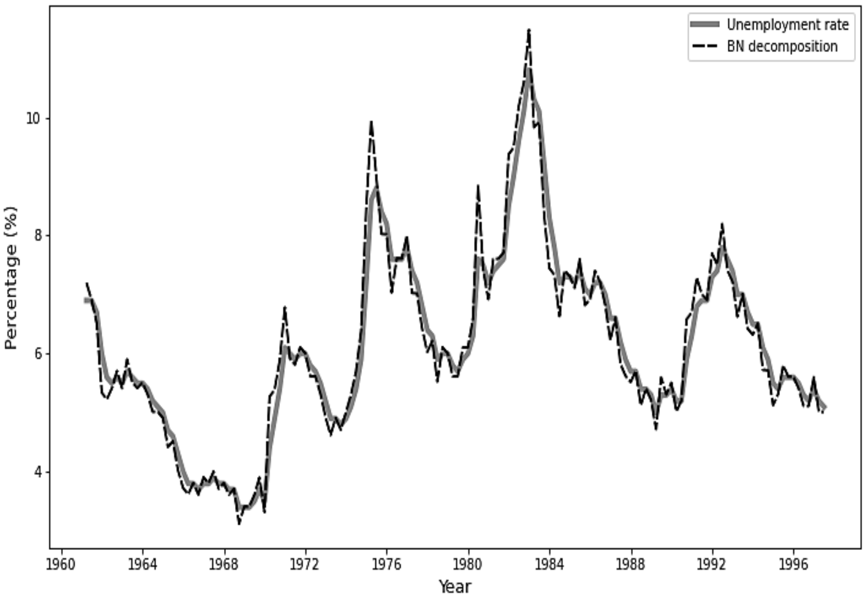
$$BN_t^\tau = u_t + \phi/(1-\phi)(\Delta u_t - \mu) \tag{9}$$

which implies that the trend is the present level of the series plus the long-run impact of the transitory momentum in the series given by the deviation of $\mathbf{u_t}$ from its steady state level. The cyclical component, BN_t^c, is obtained as the residual of $\mathbf{u_t}$ after accounting for BN_t^τ.

We use monthly data from 1960 to 1997 to estimate an AR(1) in unemployment with 8 lags. We obtain the coefficient of persistence, $\boldsymbol{\phi} = \mathbf{0.4894}$ and substitute it in equation (9) to obtain the Beveridge-

Nelson trend components. This new series gives us the steady-state component of the unemployment series, which makes the short-run natural rate estimates. The estimate of the natural rate, which is plotted together with the actual unemployment rate in Figure 5, confirms the conclusion hinted at by the impulse response functions; that fluctuations (or shocks) in the natural rate explain the bulk of fluctuations in the actual unemployment rate over time. The unemployment rate is also a smoother series around its non-stationary moving target of the natural rate, in line with several such claims in recent literature.

Figure 5: The Natural Rate of Unemployment - BN Decomposition

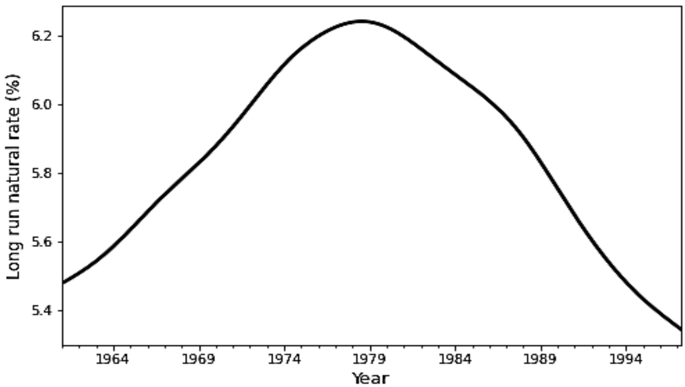


Source: Authors' own calculations

Determinants of the Natural Rate

We understand from Ireland's (1999) study that the natural rate drives the inflation and unemployment processes. We also saw that fluctuations in the natural rate explain majority of the short run variation in unemployment and modest variation in the inflation rate. We now need to examine the factors that determine the natural rate and cause shifts in the same. In we saw unusually volatile movement in the natural rate roughly between 1970 and 1980 and then between 1980 and 1984. We plot the long-run natural rate to show the trend of movement in the rate across four decades in Figure 6.

Figure 6: Long Run Natural Rate



Source: Federal Reserve Bank of St. Louis FRED

We also seek to understand if the determinants of the natural rate are structural variables of the labor market, to validate our estimates of the natural rate since we based our study on the natural rate hypothesis. We follow King and Morley (2007) in their approach to estimate determinants using some simple regressions. We examine the relationship between our short-run estimates of the natural rate and some variables emphasized by previous studies, principally those concerning labor-market search theory.[5]

Several theorists believe the natural rate to be driven by a few sets of variables; we are going to discuss them briefly and test the ones which were data-accessible.[6] Four variables that are driving factors in

[5] We largely follow discussions of variables as given by Hall (2005) for testing.

[6] For a detailed set of references on labor-market search theories, refer to King and Morley (2007).

standard models are reservation wages, labor productivity, real interest rates and worker bargaining power. Higher returns to not working, such as unemployment benefits, raise reservation wages of workers and thus should increase the natural rate. Higher productivity growth typically results in lower unemployment by increasing the surplus from employment and thus increasing the incentives facing employers to create jobs, and should thus reduce the natural rate. At the same time it is possible to obtain a positive relationship because ceteris paribus more output is being produced by the same set of workers which reduces the demand for labor. A higher real interest rate raises unemployment by decreasing the present value of the employer's part of the surplus (productivity minus wage) and reduces hiring, increasing the natural rate. Greater bargaining power in the hands of the workers reduces the firm's surplus and the incentive to hire. It can be achieved by strong union memberships and (higher) real minimum wages. Other important driving forces include real wages and vacancy rates; these variables are usually treated as endogenous by search theories. All else equal, higher real wages should reduce the firm's incentive to hire and increase the natural rate. All else equal, a permanent increase in vacancies should lower the natural rate since the demand for labor is higher. It is important to account for endogeneity when working with such variables.

There are other possibly relevant variables which aren't addressed in standard models. These include the growth rate of labor force, demographic factors (based on age, sex) and exogenous shifts in sectoral composition. These variables are believed to have longer gestation periods and relatively long-run effects on the natural rate. The rationale for growth rate of labor force is that a faster rate of workers entering the workforce may increase the frictional unemployment and thus the natural rate. Remember from our previous discussion that frictional unemployment makes the temporary component of unemployment which is why a rise in the frictional rate could possibly increase the natural rate. Shifts in sectoral composition proxies cross-sectional worker heterogeneity across industries; with changes in relative labor demand in various industries, workers must learn new skills and this shifts the industry's-share-of-employment and worker-skill dynamics, which can potentially have long-term effects for the unemployment rate.[7]

Setting up the Models

Data was borrowed from the Federal Reserve of St. Louis FRED and ALFRED databases and the Bureau of Labor Statistics for 1960Q1:1997Q4. Most of the data was available for monthly frequency[8]. Of the set of variables discussed, we treat the following variables as regressors of the natural rate:

1. Reservation wages; proxy: log real unemployment benefits per unemployed person
2. Labor productivity growth; proxy: change in log output per worker (monthly)
3. Real interest rate; proxy: CPI-deflated 10-year treasury yield
4. Real wages; proxy: log real hourly compensation
5. Growth rate of labor force (monthly)
6. Exogenous change in sectoral shifts[9] (monthly)

We have six regressors[10] and the natural rate as our dependent variable. The two underlying models we use are a simple ordinary least squares (OLS) and a two-stage-least-squares (2SLS) specification. The OLS method estimates the parameters of a linear function of a set of explanatory variables by minimizing the sum of squares of the differences between the observed dependent variable (values of the variable being observed) in the given dataset and those predicted by the linear function. In models where we have presence of endogeneity in potential

[7] Hypothesized by Lilien (1982)

[8] Any series available in quarterly form was interpolated to get a monthly series

[9] Construction: sum of absolute value of the monthly changes in percentage composition of each major employment sector- manufacturing, construction, finance, government, mining, service, transportation and utilities, retail sales, wholesale sales.

[10] A proxy for 'bargaining power'- log real minimum wage, was tested in all regressions; it had to be dropped due to its near-perfect collinearity (0.92) with 'real wages' variable.

regressors, the error terms and the regressor are correlated due to which OLS gives biased results. In such a case we can use the method of Instrumental Variables (IV) which carries out the regression using a 2SLS specification. It is carried out in two stages:

1. The first stage is the regression of the endogenous variable on the set of exogenous variables and its own instrumental variable (or proxy), using OLS, from which we derive the predicted values of the endogenous variable. This is done to incorporate the effect of the IV on the endogenous variable to mitigate the endogeneity.
2. In the second stage, we use the predicted values of the endogenous variable from stage 1 and regress the dependent variable on all exogenous variables plus the new predicted variable, using OLS. The IV is done away with at stage 2 and does not enter the final regression.

The model specifications for our six models are given in Table 3. Models I and II are regular OLS in levels for all variables plus a time trend to control for any differing drift in the series. In Models III and IV we take the first differences of all non-stationary series including the dependent variable to make the model stable; we don't difference stationary variables so as to prevent the loss of information from unnecessary differencing. We model separately for the same variables and specifications between 1970 and 1980 to isolate the effect of the stagflation without pooling data from all time periods. Many relationships turned around during this period which could give us interesting insights into how the determinants of the natural rate behaved in comparison to their behavior across the four decades.

In Model II, we let the endogenous variables be: real unemployment benefits, labor productivity growth, real hourly wages and the real 10-yr treasury yield.[11] Our instruments are twelve month lags of the endogenous variables; the lags satisfy conditions of IVs as their correlation with the natural rate is low while they are highly correlated with their endogenous counterparts.

We attempt an identical specification of Model II in Model IV and Model VI by choosing the same set of endogenous variables. However, Wu-Hausman's test of exogeneity failed to reject the null at 10 percent, to yield them as exogenous variables in the differenced model[12]; through iterations we end up with real hourly wages as our endogenous variable. We take two IVs in this model- the twelve month lag of real hourly wages, and another proxy given by 'personal consumption expenditure'[13]; this variable is again highly correlated with real wages but has little to no correlation with the natural rate, both statistically as well as intuitively. It is essential to have at least as many instruments as the endogenous variables to fully identify the system; under-identification is a problem but over-identification is not a real cause of concern.

Regression Results

OLS levels (Model I) gives the best fit out of all six models that we estimated, with an R^2 of 0.45. However, from the Durbin-Watson test statistics, both models in levels (I and II) indicate presence of positive autocorrelation of the first order (AR(1)). This may be since the natural rate is a non-stationary series, among other variables, which can also render the residuals to be non-stationary. As a consequence, the regression results for Models I and II may be spurious. Spurious

[11] King and Morley (2007) note that it is possible that variables (other than real wages which are treated as endogenous by standard theories) that are allowed to be endogenous are determined simultaneously with the natural rate. Hence, from our set of regressors, we believed these four (in model II) could be endogenous to the system. We test their exogeneity with the Wu-Hausman test where the null, H_0: All endogenous variables are exogenous, is rejected, implying that our chosen variables are indeed endogenous.

[12] p-value for the Wu-Hausman test = 0.11 which indicates a compromise in meeting the endogeneity conditions. We want a variable to satisfy the endogeneity condition and settle at 'real hourly wages' for our analysis.

[13] As far as the author is aware, this has not been included earlier in any study and since it is a good proxy for wages, it can help identify our system well by the theory of instrumental variables

regressions can have high R^2 values but incorrect signs and unreliable coefficients. However, all but the sectoral shifts variable are significant in Model I, whereas unemployment benefits and growth of labor force are significant in Model II. However, labor productivity and unemployment benefits are assigned a positive sign which is counter-intuitive as per theory and general reasoning.

The Durbin-Watson statistics from Models III and IV, which are considered in first differences, suggest that we have eradicated autocorrelation from the previous specifications. Both models suggest that unemployment benefits, labor productivity, real interest rate (given by treasury yield) and growth of labor force are significant variables in determining the natural rate of unemployment. Real wages are also significant but at lower levels of confidence; this may be because of their high standard errors. Similarly, real interest rate depicts negative signs in these two models which is something to look into. For Models V and VI[14], that is the period between 1970 and 1980, we have three significant variables in the regressions. These are unemployment benefits, growth of labor force and labor productivity (using OLS).

Table 3: Model Specifications

Model I - OLS Levels	Model IV - 2SLS First Differences
Model II - 2SLS Levels	Model V - OLS 1970-1980 First Differences
Model III - OLS First Differences	Model VI - 2SLS 1970-1980 First Differences

[14] 'Real hourly wages' satisfies the endogeneity conditions.

Table 4: Determinants of the Natural Rate of Unemployment

Variable	Model I	Model II	Model III	Model IV	Model V	Model VI
Intercept	-2.89***	-6.29***	-0.01	-0.03**	-0.02	0.02
	(0.47)	(2.29)	(0.007)	(0.01)	(0.01)	(0.05)
percent Sectoral shifts	0.05	0.02	0.006	-0.003	0.001	0.005
	(0.04)	(0.06)	(0.006)	(0.008)	(0.01)	(0.01)
Real Unemp. Benefits	0.55***	0.74***	-0.16***	-0.18***	-0.15***	-0.17***
	(0.04)	(0.08)	(0.025)	(0.03)	(0.05)	(0.05)
Labor Productivity Growth	0.12***	0.52	-0.04***	-0.06***	-0.05***	-0.02
	(0.04)	(0.39)	(0.006)	(0.01)	(0.01)	(0.03)
Real hourly compensation	-0.76***	-1.97	0.84**	3.89**	1.41*	-2.33
	(0.22)	(1.34)	(0.36)	(1.96)	(0.84)	(4.05)
Real 10 yr treasury yield	0.42***	1.43***	-0.21***	-0.21***	-0.13	-0.09
	(0.08)	(0.20)	(0.06)	(0.06)	(0.11)	(0.11)
Labor force growth rate	-0.12***	-0.07	-0.06***	-0.05***	-0.06***	-0.07***
	(0.04)	(0.11)	(0.007)	(0.007)	(0.01)	(0.01)
Time Trend	0.01***	0.03***				
	(0.002)	(0.01)				
R squared	0.445	0.215	0.331	0.222	0.398	0.293
Adjusted R squared	0.436	0.202	0.321	0.211	0.366	0.256
Durbin-Watson	0.166	0.453	2.679	2.287	2.634	2.484
Observations	438	426	437	425	120	120

Notes: Coefficients from the regression on the (BN-trend) natural rate series are reported with standard errors in parentheses (). All real quantities are measured in 1983 US dollars. ***, **, * denote significance at 1 percent, 5 percent and 10 percent levels respectively.

Consider Model IV in which we account for non-stationarity as well as endogeneity for the full sample period. We interpret the coefficients as signifying the average change in natural rate, in standard deviation terms, for a 1-SD change in the explanatory variable keeping all else constant. For instance, for a 1-SD increase in real unemployment benefits to workers, our model predicts a 0.18 SD decrease in the natural rate.

From these results, we can conclude that for data as we adapted and the models that we fit, the variables which explain variation in the natural rate more significantly than others are: real unemployment benefits (reservation wages), labor productivity growth (productivity), real ten-year treasury yield (real interest rate), real hourly compensation (real wage) and labor force growth rate. Our results slightly contrast those from a similar regression exercise by King and Morley (2007), since they find sectoral shifts to be significant in place of real interest rate and labor force growth rate. Our sign on labor force growth rate is negative across the board which might deny the 'frictional unemployment' explanation and suggest that a faster influx of workers in the labor-force simply brings down unemployment and the natural rate in the longer run.

Potential Historic Factors

Data indicates two major shifts in the natural rate- one during the 1960s and another in the 1980s. The 60s and 70s were a period of increase in the natural rate due to changes in labor market institutions that affected the shape of the supply curve more than changes that affected labor demand.

As we discussed earlier in the section, the supply of labor is partly determined by the reservation wage of potential workers, which in turn is shaped by institutions like the minimum wage, the level and duration of unemployment benefits, social security payments etcetera. A rise in the influence of labor unions and labor militancy during this time increased the reservation wages, leading to a rise in the natural rate in the US (Srinivasan and Mitra 2012). There was also an increase in labors' income share, which we know to positively affect the natural rate. In early 1970s, the world price of energy soared, causing a fall in productivity, which as we found through regressions, would contribute to a rise in the natural rate. The world real interest rate increased in the 80s with the spike in federal funds rate by the US to control high inflation. In Figure 5 we see another spike in the natural rate starting in 1980, and we also learned how high interest rates increase the natural rate levels.

A decline in the natural rate began in the 90s which was an era marked by relatively weak labor unions, low minimum wage and a slight decline in labor's income share (Phelps 1998). Gordon (1997) also attributes the lowering of the natural rate in the 90s to the growing share of computer output in both GDP and personal consumption. This can be attributed to increases in productivity levels which we know to be negatively correlated with the natural rate. These trends across the decades allow us to map the variations in the natural rate to structural factors that we discussed earlier.

In light of our discussion of the natural rate, its effects on the economy and movement of its own, King and Morley (2007) make an interesting observation- while movements away from the steady state are governed by a strong Phillips curve relationship, a sizeable proportion of macroeconomic activity is governed by changes in the steady state (natural rate) itself, even over short horizons. This is also supported by Staiger (2001) who notes that the slope of the Phillips curve has been quite stable over time but the level of the unemployment rate that is consistent with stable inflation viz. the natural rate has shifted.

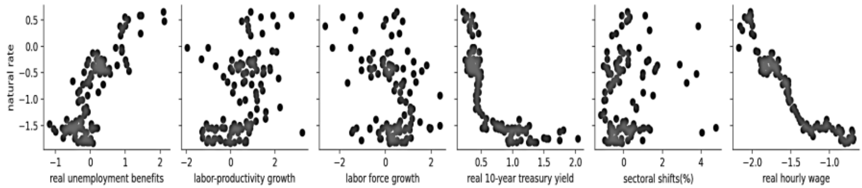
Further Work

There are several avenues for extending work presented here. We list a few of them in this section.

The Beveridge-Nelson decomposition used to estimate the short-run natural rate was based on a univariate time-series in our work. This can be extended to a multivariate system which captures greater features of the economic system to get richer estimates.

In Figure 7 below, we obtain pair-plots of the natural rate against some of its determinants: real unemployment benefits, labor productivity growth, labor force growth, real ten-year treasury yield, percentage changes in sectoral shifts and real hourly wage, from left to right. We plot these relationships across time, for the years 1960-1969, 1970-1979 and 1980-1997. As we can see, some of these variables show strong correlation across time, with the natural rate. However, they show significant variation in the strength of these relationships.

Figure 7: Relationships between Natural Rate and Significant Determinants



a. 1960 - 1969

b. 1970 - 1979

c. 1980 - 1997

In the previous section, we conducted regressions across the entire time period of 1960 to 1997. Such pooled regressions give the dataset more variability at the risk of ignoring interesting period-specific relationships. We conducted linear regressions on our data, but the graphs above also depict more than just linear relationships, which can be better captured by some non-linear specifications.

APPENDIX

A. Solving the Barro-Gordon Model

The B-G model is defined by the following three fundamental equations:

$$U_t = U_t^n - \alpha(\pi_t - \pi_t^e) \tag{A.1}$$

where $\alpha > 0$.

$$U_t^n - U_{t-1}^n = \lambda(U_{t-1}^n - U_{t-2}^n) + \varepsilon_t \tag{A.2}$$

where $-1 < \lambda < 1$ and $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ is serially uncorrelated.

$$\pi_t = \pi_t^p + \eta_t \tag{A.3}$$

where $\eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$ is serially uncorrelated and covariance $\sigma_{\eta\varepsilon}$ with ε_t.
The government's loss function is

$$(1/2)(U_t - kU_t^n)^2 + (b/2)\pi_t^2 \tag{A.4}$$

On applying the RE operator to (A.4), the optimization problem becomes

$$\min_{\pi_t^p} E_{t-1}\{(1/2)[(1-k)U_t^n - \alpha(\pi_t^p \eta_t - \pi_t^e)]^2 + (b/2)(\pi_t^p + \eta_t)^2\} \tag{A.5}$$

First-order-condition of the loss minimization problem:

$$\alpha E_{t-1}[(1-k)U_t^n - \alpha(\pi_t^p - \pi_t^e + \eta_t)] = bE_{t-1}[\pi_t^p + \eta_t] \tag{A.6}$$

Solving for π_t^p and letting $\pi_t^p = \pi_t^e$; also assuming that the expected value of a shock in the next time period is zero i.e. $E_{t-1}\eta_t = 0$, we get

$$\pi_t^p = \pi_t^e = (1 - k/b)\alpha E_{t-1}U_t^n$$

Let $(1 - k/b) = A > 0$, so that we have the following relation in equilibrium:

$$\pi_t^p = \pi_t^e = \alpha A E_{t-1}U_t^n \tag{A.7}$$

At $\pi_t = \pi_t^e = 0$, the marginal benefit from creating inflation surprise exceeds its marginal cost (which turns out to be equal to $\pi_t = 0$). Thus, the policymaker would increase the planned rate of inflation beyond zero which proves his clear incentive to demonstrate time-inconsistency in monetary policy.

The mean inflationary bias can be derived by taking the expectation on both sides of (A.7) and is given by

$$E(\pi_t) = \alpha A E_{t-1} U_t^n \tag{A.8}$$

The equilibrium inflation rate co-moves with the natural rate of unemployment. From equations (A.1), (A.3) and (A.7), we get

$$U_t = U_t^n - \alpha \eta_t \tag{A.9}$$

To get the equilibrium expressions of the two rates we substitute (A.2) in (A.8) and get

$$U_t = \lambda \Delta U_{t-1}^n + U_{t-1}^n - \alpha \eta_t + \varepsilon_t, \tag{A.10}$$

From (A.3), (A.7), (A.8) and (A.9), we get the equilibrium inflation rate

$$\pi_t = \alpha A U_{t-1}^n + \alpha A \lambda \Delta U_{t-1}^n + \eta_t \tag{A.11}$$

To test for *cointegration*, we construct a linear combination using (A.10) and (A.11), and get

$$\pi_t - \alpha A U_t = -\alpha A \varepsilon_t + (1 + \alpha^2 A)\eta_t, \tag{A.12}$$

To understand the short-run dynamics, we impose some restrictions. Taking first differences of (A.9) and solving for $\Delta U_t^n = U_t^n - U_{t-1}^n$, we get

$$\Delta U_t^n = \Delta U_t + \alpha(\eta_t - \eta_{t-1})$$

Substituting this result in (A.2), we obtain the stationary change in unemployment:

$$\Delta U_t = \lambda \Delta U_{t-1} + \varepsilon_t - \alpha \eta_t + \alpha \eta_{t-1}(1 + \lambda) - \alpha \lambda \eta_{t-2} \tag{A.13}$$

Writing (A.12) and (A.13) in the form of a vector ARMA (1, 2) for the (a) stationary linear combination of inflation and unemployment and (b) the stationary change in unemployment, we have

$$\begin{aligned}\begin{bmatrix} \pi_t - \alpha A U_t \\ \Delta U_t \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} \pi_{t-1} - \alpha A U_{t-1} \\ \Delta U_{t-1} \end{bmatrix} + \begin{bmatrix} -\alpha A & 1 + \alpha^2 A \\ 1 & -\alpha \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ 0 & \alpha(1+\lambda) \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1} \\ \eta_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & -\alpha\lambda \end{bmatrix} \begin{bmatrix} \varepsilon_{t-2} \\ \eta_{t-2} \end{bmatrix}\end{aligned} \tag{A.14}$$

B. Phillips Perron Test

The null hypothesis for the PP-test is that the series contains a unit and rejection of the null implies trend stationarity in the time series.

Table B.1: Phillips-Perron Test

	ρ	τ	q	Z_τ
1960:1 - 1997:2				
Unemployment rate	0.9743	-1.3627	5	-2.2161
Inflation rate	0.8928	-2.8739	0	-2.8739**
1974:1 - 1997:2				
Unemployment rate	0.9586	-1.3417	4	-2.1205
1981:4 - 1997:2				
Inflation rate	0.6166	-4.3842	0	-4.3842**

Notes: ρ: slope coefficient; τ: t-statistic; Z_τ : P-P statistic
Critical value of PP-test statistic at 5 percent level of significance = **-2.87**
** Denotes significance at the 5 percent level.

Table B.1 shows estimates of ρ, the slope coefficient from a regression of each variable on a constant and its own lagged value, as well as τ, the conventional t-statistic for testing the hypothesis that $\rho = 1$. The Phillips-Perron statistic, Z_τ, adjusts the conventional t-statistic to allow for serial correlation in the regression error. This adjustment uses Newey and West's method to estimate the variance of the regression error and Andrews' method to select a value for the lag truncation parameter q required to form the Newey West estimator, assuming that the process for the regression error is well-approximated by a first-order autoregression.[15]

C. Impulse Response Functions

Figure C.1 depicts the IRF of unemployment in levels to the NRU shock, 30 periods ahead. The effect of a 1-SD positive shock in NRU dissipates in the long-run, implying no structural effect of the NRU on

[15] The value for the Newey-West lag truncation parameter, q has been borrowed from Ireland (1999). For more on the PP-test for this problem, refer to the same.

unemployment. However, the standard error bands are quite wide and do not converge in the long-run, taking away some reliability from the results.

Figure C.1: IRF of Unemployment to NRU Shock - Levels Series

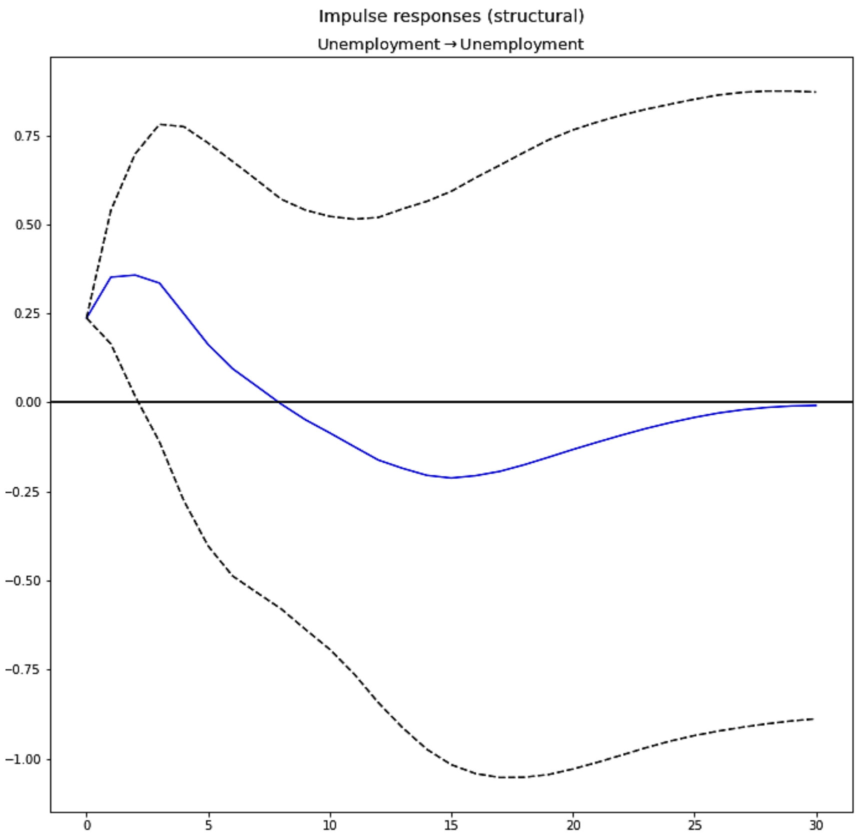
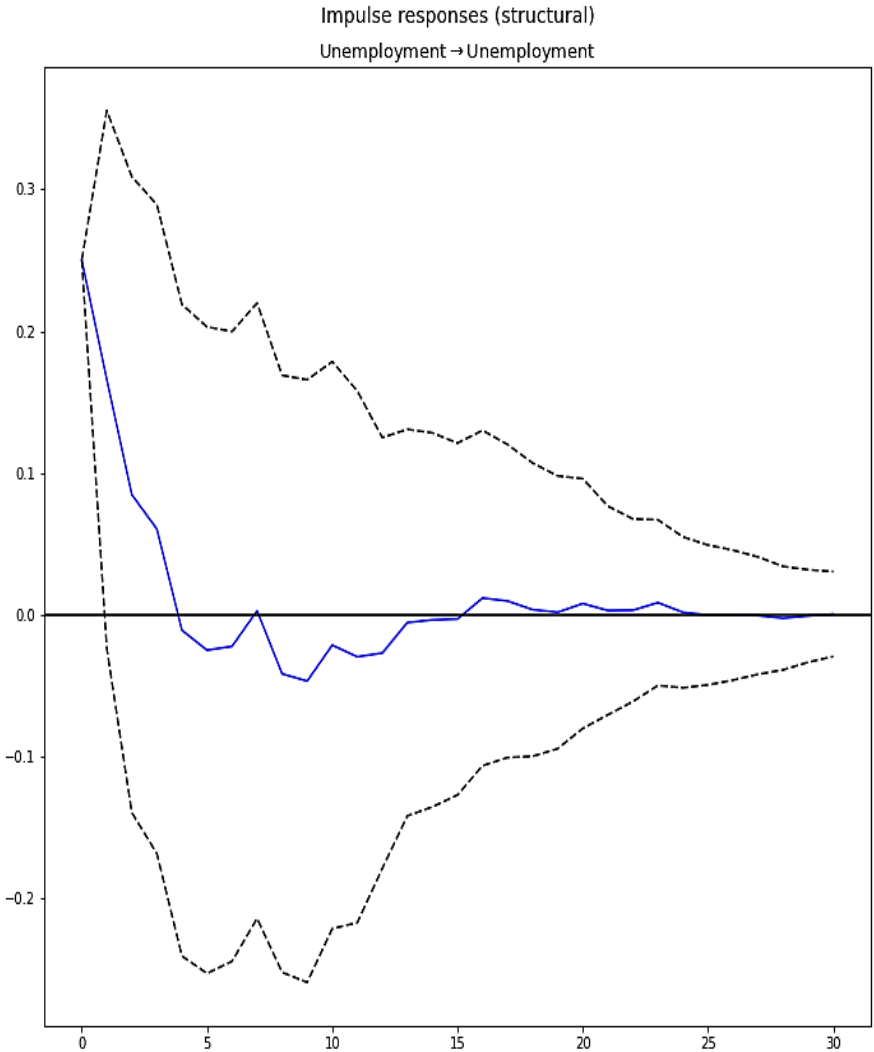


Figure C.2 depicts the IRF of twice-differenced unemployment (to induce stationarity) to the NRU shock, 20 periods ahead. The effect of a 1-SD positive shock in NRU dissipates in the long-run, implying no structural effect of the NRU on unemployment. However, the standard error bands converge in the long-run, rendering reliability to our results.

Figure C.2: IRF of Unemployment to NRU Shock - Stationary Series



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