Calendar Anomaly and the Degree of Market Inefficiency of Bitcoin

S. Raja Sethu Durai
Sunil Paul

MADRAS SCHOOL OF ECONOMICS
Gandhi Mandapam Road
Chennai 600 025
India
April 2018
Calendar Anomaly and the Degree of Market Inefficiency of Bitcoin

S. Raja Sethu Durai
Associate Professor
School of Economics, University of Hyderabad, Hyderabad-500046, India
rdurai@uohyd.ac.in

and

Sunil Paul
Assistant Professor, Madras School of Economics
sunil@mse.ac.in
Calendar Anomaly and the Degree of Market Inefficiency of Bitcoin

S. Raja Sethu Durai and Sunil Paul

Abstract

We find evidence for a day of the week calendar anomaly in Bitcoin returns and argue that not considering this anomaly results in a biased estimate of the market efficiency. Further, it also quantifies the degree of market inefficiency of Bitcoin over time using the methodology proposed by Ito and Sugiyama (2009) and found evidence in favor of very high degree of market inefficiency for the period spanning from July 2010 to January 2018. This result contradicts some of the recent studies that argue in favor of the market efficiency of Bitcoin.

Key words: Bitcoin, Level of inefficiency, State-space model.

JEL Codes: G14, C32
INTRODUCTION

In recent times crypto-currencies particularly Bitcoin received much attention academically. For example, Dyhrberg (2016a, b) and Bouri et al. (2017a, b) examined the usefulness of Bitcoin as headge and Nadarajah and Chu (2017), Bariviera (2017) and Tiwari et al. (2018) examined the market efficiency of its price movements. All these studies reported results in favor of Bitcoin as a usable hedge against inflation, other assets, and even uncertainty while on the efficiency front, in general, are in favor of market efficiency of Bitcoin. Even the studies that identified market inefficiency for a particular sample period [Urquhart (2016)] report that the market is tending towards efficiency in the latter time period. Recently, Caporale and Plastun (2017) found evidence in favor of calendar effect in the returns with a very strong day of the week effect in the returns of Bitcoin prices, where the returns on Mondays are higher than other days of the week indicating market inefficiency.

Efficient Market Hypothesis (EMH) of Fama (1970) is the basis for all the studies on the market efficiency of the returns for an asset. The widely tested weak form of EMH implies that past information is not useful in predicting future returns, in other words, returns over a period of time are independent of each other. The simplest test to examine this weak form of EMH is to fit a correlogram of the returns in which if you find autocorrelation for various lags indicating inefficiency of the market and vice versa. Further, there can also be a case of short-term/long-term autocorrelation, where the returns are autocorrelated only for immediate/farther lags and not for other lags, either way, the case for understanding the degree of market inefficiency over time arises only when the returns are autocorrelated.

1 Similarly Cheah and Fry (2015) observed that Bitcoin exhibits speculative bubbles and the fundamental price of Bitcoin is zero.
This study is aimed at understanding the calendar anomaly particularly the day of the week effects in the daily returns of Bitcoin prices. Further, it also quantifies the degree of market inefficiency of Bitcoin over time using the methodology proposed by Ito and Sugiyama (2009). The main contribution of this study is in twofold; first, almost all the studies examining the market efficiency of Bitcoin prices used daily returns data without analyzing the autocorrelation properties of it, we establish that in the presence of short-term/long-term autocorrelation and day of the week effect, daily returns data provides a biased estimate of the market efficiency; second, most of the studies in the empirical literature have examined the question of market efficiency or inefficiency of Bitcoin prices but no study to our knowledge quantifies the degree of market inefficiency in order to understand the dynamic nature of it. The rest of the study is organized as follows: Section 2 discusses the data along with the methodology to identify calendar effect and a brief description of Ito and Sugiyama (2009) methodology. Section 3 provides the results of the empirical analysis and the final section concludes.

**DATA AND METHODOLOGY**

Following other studies, we also used the daily data on closing prices of Bitcoin collected from www.coindesk.com for the period from 18 July 2010 to 11 January 2018 and calculated return \( R_t \) as the natural logarithmic first difference of the closing prices with 2734 observations.

**Measuring the Calendar Effect**

We adopted a rolling regression approach of Zhang and Lin (2017) to identify the day of the week effect for the returns as follows:

\[
R_t = \sum_{i=1}^{7} \gamma_i D_{it} + \varepsilon_t
\]  

where \( D_{it} \) is a dummy variable that takes the value 1 for the \( i^{th} \) day of the week and 0 otherwise; since Bitcoin is traded all the days of the week, we have 7 dummy variables to denote each day of the week and \( \gamma_i \) is the
corresponding parameter estimated for \(i\)th day of the week. In a standard regression inference if the t-values of this parameters are greater than 1.96 it implies a prominently significant day of the week effect at 5 percent level. Further, a rolling regression estimation of equation (1) with a window size \(n\) is used to understand the stability of the calendar anomaly and to realize the magnitude of the anomaly, for each window size, we calculate the number of times the t-values of each parameter \(\gamma_i\) in absolute terms exceeds 1.96 as a proportion to the total number of regression estimated\(^2\).

**The Degree of Market Inefficiency**

The state space model proposed by Ito and Sugiyama (2009) to quantify the degree of market inefficiency over time is based on Kalman filter technique, which is used to extract the random fluctuations concealed in the data; by combining the information from the observation equation (prediction) and transition equation (measurement) gives the best estimate of it. The model is specified as follows and estimated using the technique developed by Ito (2007) to derive Kalman smoothing:

*Observation Equation:*

\[
R_t = (R_{t-1}, R_{t-2}, \ldots, R_{t-k}) \begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \vdots \\ \beta_{kt} \end{pmatrix} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma_{\varepsilon t}^2) \tag{2}
\]

*Transition Equation:*

\[
\begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \vdots \\ \beta_{kt} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} \begin{pmatrix} \beta_{1t-1} \\ \beta_{2t-1} \\ \vdots \\ \beta_{kt-1} \end{pmatrix} + \begin{pmatrix} \vartheta_{1t} \\ \vartheta_{2t} \\ \vdots \\ \vartheta_{kt} \end{pmatrix} ; \vartheta_t \\
\equiv \begin{pmatrix} \vartheta_{1t} \\ \vartheta_{2t} \\ \vdots \\ \vartheta_{kt} \end{pmatrix} \sim N_k(0, \sigma_{\vartheta t}^2 I) \tag{3}
\]

\(^2\)For the data with \(T\) observations, the total number of regression estimated in a rolling regression with \(n\) window size is equal to \((T-n)+1\)
RESULTS

We estimate a rolling regression of the equation (1) for a window size of 1000 with a total number of 1735 regressions and the t-values of each day parameter is depicted in Figure 1. The figure clearly shows that there is a consistent and significant day of the week effect for Mondays and Tuesdays while smaller effect is seen for Wednesdays and Thursdays. The same is also inferred from Table 1 where the proportion of significant t-values of each day parameter out of 1735 regressions is presented. This provides an indicative evidence for both the day of the week effect as highlighted by Caporale and Plastun (2017) in Bitcoin returns and market inefficiency.

![Figure 1: Rolling regression t-values](image)

<table>
<thead>
<tr>
<th>Day</th>
<th>M</th>
<th>T</th>
<th>W</th>
<th>T</th>
<th>F</th>
<th>Sa</th>
<th>Su</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion in percent</td>
<td>53.43</td>
<td>45.53</td>
<td>12.91</td>
<td>10.14</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Source: Author’s own calculation.

In this study, we compute Bitcoin return as $R_{1t} = \ln\left(\frac{P_t}{P_{t-1}}\right)$ and to address the autocorrelation of the daily returns from the fifth lag, and to
incorporate the day of the week effects, two additional variants of returns also calculated as $R_{5t} = \ln\left(\frac{P_t}{P_{t-5}}\right)$ (5-day return) and $R_{7t} = \ln\left(\frac{P_t}{P_{t-7}}\right)$ (7-day return); where $P_t, P_{t-1}, P_{t-5}$ and $P_{t-7}$ are the daily closing prices of Bitcoin prices at time period $t$, $t-1$, $t-5$ and $t-7$, respectively. The Q-statistic from the autocorrelation function of $R_{1t}$ indicates a long-term autocorrelation of the daily returns with no autocorrelation in the first four lags but from the fifth lag, there is autocorrelation. This itself indicates a time-varying market efficiency of the Bitcoin prices. The autocorrelation function of these later two return series ($R_{5t}$ and $R_{7t}$) are totally different with autocorrelation in all the lags compared to the former return series (Table 2). All the three return series are used to quantify the degree of market inefficiency using the model proposed by Ito and Sugiyama (2009). The results for a model with one lag and are presented in the Figures 2, 3 and 4 where, the β values close to zero is considered as market efficiency and closer to 1 as market inefficiency. All the figures indicate a time-varying market inefficiency.

### Table 2: Ljung Box Q Statistics for Autocorrelations

<table>
<thead>
<tr>
<th></th>
<th>$R_{1t}$</th>
<th>$R_{5t}$</th>
<th>$R_{7t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(1)</td>
<td>2.23</td>
<td>1811.01</td>
<td>2003.61</td>
</tr>
<tr>
<td></td>
<td>(0.14)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Q(2)</td>
<td>3.28</td>
<td>2888.27</td>
<td>3410.09</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Q(3)</td>
<td>3.31</td>
<td>3424.14</td>
<td>4374.30</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Q(4)</td>
<td>3.75</td>
<td>3602.12</td>
<td>4993.62</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Q(5)</td>
<td>14.90</td>
<td>3620.82</td>
<td>5374.06</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Q(10)</td>
<td>47.98</td>
<td>3738.34</td>
<td>5730.60</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Q(15)</td>
<td>63.88</td>
<td>3953.74</td>
<td>6071.75</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

**Source:** Author's own calculation.

**Note:** P values are reported in (#)
The degree of market inefficiency calculated for the daily returns and depicted in Figure 2 indicate a fluctuation around zero with the mean value around 0.02, this is mainly due to many negative values in the estimated $\beta$ that bears no meaning in quantifying the degree of market inefficiency, since the values of $\beta$ ranges from 0 to 1. While the values of $\beta$ greater than 1 can be considered as an explosive behavior of the return, the negative values indicate a bias in the estimation. The degree of market inefficiency calculated for the 5-days return and 7-days return are having mean values of 0.733 and 0.831 respectively indicating higher degree of market inefficiency. In both the return series, the estimated $\beta$ exceeded the value of 1 during the years 2011 and 2013 signifying a possible explosive behavior. Altogether the results suggest a tenacious degree of market inefficiency prevails in the Bitcoin market.
Figure 3: Degree of Market Inefficiency for 5-days Return

Figure 4: Degree of Market Inefficiency for 7-days Return
CONCLUSION

This study finds evidence for day of the week effect supporting the argument of calendar anomaly in Bitcoin returns and confirms that not considering this anomaly will result in a biased estimation of market inefficiency. Further, it also establishes that a higher degree of market inefficiency persists in Bitcoin returns.
REFERENCES


MSE Monographs

* Monograph 30/2014
  Counting The Poor: Measurement And Other Issues
  C. Rangarajan and S. Mahendra Dev

* Monograph 31/2015
  Technology and Economy for National Development: Technology Leads to Nonlinear Growth
  Dr. A. P. J. Abdul Kalam, Former President of India

* Monograph 32/2015
  India and the International Financial System
  Raghuram Rajan

* Monograph 33/2015
  Fourteenth Finance Commission: Continuity, Change and Way Forward
  Y.V. Reddy

* Monograph 34/2015
  Farm Production Diversity, Household Dietary Diversity and Women’s BMI: A Study of Rural Indian Farm Households
  Brinda Viswanathan

* Monograph 35/2016
  Valuation of Coastal and Marine Ecosystem Services in India: Macro Assessment

* Monograph 36/2017
  Underlying Drivers of India’s Potential Growth
  C. Rangarajan and D.K. Srivastava

* Monograph 37/2018
  India: The Need for Good Macro Policies (4th Dr. Raja J. Chelliah Memorial Lecture)
  Ashok K. Lahiri

* Monograph 38/2018
  Finances of Tamil Nadu Government
  K R Shanmugam

* Monograph 39/2018
  Growth Dynamics of Tamil Nadu Economy
  K R Shanmugam
Calendar Anomaly and the Degree of Market Inefficiency of Bitcoin

S. Raja Sethu Durai
Sunil Paul

MADRAS SCHOOL OF ECONOMICS
Gandhi Mandapam Road
Chennai 600 025
India

April 2018